Stability of the lattice formed in first-order phase transitions to matter containing strangeness in protoneutron stars.

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Well into the deleptonization phase of a core collapse supernova, a first-order phase transition to matter with macroscopic strangeness content is assumed to occur and lead to a structured lattice defined by negatively charged strange droplets. The lattice is shown to crystallize for expected droplet charges and separations at temperatures typically obtained during the protoneutronstar evolution. The melting curve of the lattice for small spherical droplets is presented. The one-component plasma model proves to be an adequate description for the lattice in its solid phase with deformation modes freezing out around the melting temperature. The mechanical stability against shear stresses is such that velocities predicted for convective phenomena and differential rotation during the Kelvin-Helmholtz cooling phase might prevent the crystallization of the phase transition lattice. A solid lattice might be fractured by transient convection, which could result in anisotropic neutrino transport. The melting curve of the lattice is relevant for the mechanical evolution of the protoneutronstar and therefore should be included in future hydrodynamics simulations.

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I. INTRODUCTION

In the past years, considerable effort has gone into the study of the existence of phases of strange matter in neutron stars [1, 2, 3, 4, 5, 6]. A macroscopic strangeness content is predicted to be energetically favorable at densities well beyond the saturation density of symmetric nuclear matter (hereafter ρ_0). Conditions with sufficiently high density and low enough electron chemical potential for the formation of a macroscopic content of strange quarks (charge $= -1/3$) or particles containing such are obtained in the interior of a protoneutronstar (hereafter PNS) after post-bounce times of several seconds. The three possible forms for the macroscopic manifestation of strangeness suggested are a K^- condensate [2, 3], the formation of hyperons [4] and deconfined quark matter including strange quarks [5, 6]. The equation of state of matter at densities $\rho > \rho_0$ is not well enough known at this point to determine which of these scenarios will actually happen and what the order of the associated phase transition is.

The present study limits itself to a certain class of scenarios which is common to all possible forms of strangeness in high density nuclear matter and which might be subject to experimental verification. I assume the phase transition to be first order and to result in the formation of a lattice in the coexistence region with different energy and charge densities of the strange and non-strange phases, as was recently predicted by various authors [7, 8, 9, 10, 11, 12]. Typical results for the parameters of the mixed phase lattice, such as the size and spatial separation and the energy and charge densities are quoted from these authors. Whereas all studies on this lattice to date are limited to cold, deleptonized neutron stars, my focus is on PNS's in the Kelvin-Helmholtz cooling phase, $\sim 1\,\mathrm{s} - 30\,\mathrm{s}$ post-bounce, for temperatures up to tens of MeV. In particular, the present paper presents the mechanical properties of the structured mixed strange/non-strange phase as they are relevant to the further PNS evolution and observational verification.

Many initial studies predicted phenomena caused by strangeness to be confined to a very small central region of the PNS [13, 14], which would most likely render their experimental verification impossible. However, Glendenning showed in 1992 [7] that the previous studies were too simplistic in their assumptions about the thermodynamic model of the PNS. In particular, there are two conserved charges in PNS matter, the baryon number and the electric charge and, therefore, two independent chemical potentials, μ_e and μ_B. During a phase transition, their equality between the strange (subscript s) and non-strange (subscript n) phases, together with similar conditions for temperature and pressure, comprise the Gibbs conditions [7]:

$$\begin{aligned} \mu_{e,s} = \mu_{e,n}; \quad \mu_{B,s} = \mu_{B,n}; \\ T_s = T_n; \quad P_s = P_n. \end{aligned} \tag{1}$$

Previously, charge neutrality had been enforced in both phases of a first-order phase transition separately, and a Maxwell construct had been used for the mixed phase: $\rho_{mixed} = \chi\rho_s + (1-\chi)\rho_n$ (χ being the volume fraction of the strange phase), where the densities of each phase (ρ_s, ρ_n) were kept constant throughout the phase transition. This additional constraint lead to an overestimate of the required density for a phase transition to deconfined (three flavor) quark matter of up to $7 \times \rho_0$ as opposed to a predicted range of $\sim 2-3 \times \rho_0$ [7, 8] with the correct treatment of the Gibbs condition. The microscopic stability condition for neutron stars $\delta P/\delta\rho \geq 0$ [15] and the constant densities in both phases also lead

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to a coexistence region consisting of an infinitely thin spherical shell with a pressure discontinuity, as opposed to a spatially extended mixed phase with continous $P(r)$-dependence which follows from the Gibbs conditions [10].

The driving force for a net exchange of charge between the strange and non-strange phases is the isospin restoring force which leaves the strange phase with a net negative charge. Two phases with opposite charges assume a spatial order which is determined by a minimal sum of the surface, curvature and Coulomb energies [9, 10]. The predicted geometry of the strange phase has been found [10, 11] to vary from spheres via rods to platelets immersed in the majority non-strange phase, as density increases. As soon as $\chi > 0.5$, the two phases reverse roles. The spatial extent of the crystalline phases depends sensitively on the neutron star mass [10] and properties of matter at supernuclear densities which are not well known, but can reach several km. Some of these properties have been investigated in recent works, such as the surface energy between the strange and hadronic phases and the effective MIT bag constant in the case of a phase transition to deconfined quark matter [12] and the surface and curvature energies between normal nuclear matter and a phase with a K^--condensate [16]. In both cases, the results were shown to be model-dependent without a definitely reliable result.

The above quoted range of $\sim 2-3\times\rho_0$ [7, 8] for the transition density is based upon the bulk approximation for both phases, which neglects any screening effects across the interface. More recent studies of surface effects, such as a transition layer with a finite thickness of $\Delta R \sim 5\,\mathrm{fm}$ due to Debye screening effects and the discontinous pressure due to the surface tension between both phases, however, show a net increase in the bulk energy density of the strange phase in the case of a K^- condensate [17], making finite-size droplets for radii smaller than $R_S \sim 10\,\mathrm{fm}$ energetically less favorable. Debye screening lengths have been reported for K^- condensed matter ($\lambda_{K^-} \sim 5\,\mathrm{fm}$ [17]), for deconfined quark matter ($\lambda_q \sim 5\,\mathrm{fm}$ [9]) and for hadronic neutron star matter ($\lambda_{D,n/p} \sim 10\,\mathrm{fm}$ and $\lambda_{e^-} \sim 13\,\mathrm{fm}$ [9]). Properly taking into account screening to find the minimum energy configuration, see [17] for K^- condensates and [18] for deconfined quark matter, effectively opens up another degree of freedom, in itself lowering the energies of both phases, which is favorable for the formation of a lattice. However, screening can also increase the electron fraction in the hadronic phase by pushing the electrons away from the negative charge on the strange droplet surface, leading to a higher negative charge concentration outside the screened region which will push the pressure at which global charge neutrality can be attained to higher values [17]. The screened charges have also been shown to increase the effective surface tension $\sigma = \sigma_{strong} + \sigma_C$ by a Coulomb contribution, making droplets with radii smaller than the Debye screening length energetically less favorable (see [18] for deconfined quark matter), thus increasing the required PNS density for the first-order phase transition. A final answer is not possible unless the exact equation of state for hadronic PNS matter with and without a K^- condensate and deconfined quark matter with strange quarks is known. This includes hyperon formation, following which a similar structured phase has not been studied to date.

Standard PNS models predict central densities in the relevant range of a few times ρ_0 after neutrinos have carried away the bulk of the lepton number in the PNS. A mixed strange/non-strange phase which might form then was shown to have a significant impact on the neutrino transport properties, resulting in a neutrino opacity up to two orders of magnitude higher for typical neutrino energies $\sim 10\,\mathrm{MeV}$ [19]. This opens up a window of observation in supernova neutrino detectors through the remaining lepton number which will be carried away by neutrinos. A pure strange phase which might form for high enough central densities would have a lower neutrino opacity compared to non-strange PNS matter [20], effectively creating a transparent strange core surrounded by a relatively opaque coexistence layer.

In the following, I will refer to a structured mixed layer as phase transition lattice, independent upon the specific spatial ordering and solid or liquid state. The temperature behavior of the phase transition lattice, including the coexistence curve between a solid, crystalline phase transition lattice and a liquid phase of the droplets containing strangeness is derived in section II. The mechanical stability of the lattice to shear stresses and its possible breakup is investigated in section III. Section IV concludes the study and gives an outlook on some possible observational signatures of the formation of the phase transition lattice.

II. THE PHASE TRANSITION LATTICE AT FINITE TEMPERATURES

A. Phase Transition Lattice Melting Curve

Because a solid, crystalline phase might have to be taken into account as a new element in hydrodynamical PNS evolution studies and since its neutrino transport properties might be different from a liquid mixed phase, it is important to know the solid-liquid coexistence curve of the phase transition lattice. At the onset of the first-order phase transition and on the outermost layers of the mixed phase in the cold neutron star, the volume fraction of the strange phase will be small. In that limit, the minority phase at zero temperature can be approximated as a Coulomb lattice of negative point charges immersed in a slightly positively charged background of normal PNS matter. The absolute charge density in the strange phase will be higher because the majority phase can significantly lower its isospin by pushing negative charge into the minority phase [12], more than compensating for the opposing effect of a higher repulsive Coulomb energy within the latter. The reverse is true for the deepest

layers of the mixed phase, where the normal PNS matter is the minority phase and a high positive charge density resides in that phase, whereas the Coulomb interaction is compensated by the condensation energy of hadrons.

The droplet charges given are to be understood as effective values, already taking into account screening effects [9, 17, 21]. Given rigid strange droplets with charge densities large compared to the surrounding hadronic PNS matter, the lattice can then be regarded as a one-component plasma (OCP). Its treatment in the harmonic approximation is well-established [22, 23, 24, 25]. The equation of motion of the α-component of the displacement u on a lattice site l for a Bravais lattice (defined as having one particle per unit cell) can be written as

$$M\ddot{u}_\alpha(l) = -\frac{\delta\Phi}{\delta u_\alpha(l)} = -\sum_{\beta l'} \frac{\delta^2\Phi}{\delta u_\alpha(l)\delta u_\beta(l')} u_\beta(l'), \quad (2)$$

where Φ is the electrostatic potential and the displacement amplitude u_α is

$$u_\alpha = \sqrt{\frac{\hbar}{2NM}} \sum_{\vec{k},j} \frac{e_\alpha(\vec{k}j)}{\sqrt{\omega_j(\vec{k})}} e^{i\vec{k}\vec{x}(l)} A_{\vec{k}j}, \quad (3)$$

with $A_{\vec{k}j} = a^\dagger_{-\vec{k}j} + a_{\vec{k}j}$ and the usual definitions for creation- and annihilation operators. The three characteristic polarization modes, two transverse and one longitudinal, of the Bravais lattice are denoted by the index j.

In this formalism, the mean square displacement relative to the distance between nearest neighbors $d = (3\pi^2)^{1/6}a$ for a BCC (body-centered cubic) lattice is

$$\begin{aligned}\frac{\langle u^2\rangle}{d^2} &= \frac{1}{d^2}\frac{\hbar}{2M}\sum_{\vec{k}j}\frac{coth(\beta\frac{\hbar}{2}\omega_j(\vec{k}))}{\omega_j(\vec{k})} \\ &= \frac{1}{d^2}\frac{3\hbar}{2M\alpha\omega_p}(1+\frac{4}{\alpha\eta}D_1(\alpha\eta)), \quad (4)\end{aligned}$$

where η is the degeneracy parameter and the Debye integral is defined as

$$D_n(x) = \frac{n}{x^n}\int_0^x dt(\frac{t^n}{e^t-1}). \quad (5)$$

The dispersion relation for the transverse modes has the acoustic Debye form [24, 25]

$$\omega_T(\vec{k}) = \alpha\omega_p\frac{k}{k_D} \quad (6)$$

with the Debye wavenumber $k_D = (6\pi^2 N/V)^{1/3}$, the plasma frequency $\omega_p = (Ze/\epsilon_0 \times N/MV)^{1/2}$ and $\alpha = 0.393$. For the longitudinal branch, the Einstein model has been suggested with a constant frequency of $\omega_L \propto \omega_p$ [24]. However, for a cubic lattice in the harmonic approximation, the symmetry condition $\langle u^2\rangle = 3\times\langle u_T^2\rangle = 3\times\langle u_L^2\rangle$ [22] makes only one branch necessary to calculate the average square displacement amplitude. For typical lattice constants of $a \approx 10\,\mathrm{fm}$, we obtain $k_D \approx 0.4\,\mathrm{fm}^{-1}$ and $\hbar\omega_p \approx 5.8\,\mathrm{MeV}$. Typical values for the strange (minority) phase were used here, a charge density of $\rho_C = 0.4\,\mathrm{fm}^{-3}$, a mass density of $\rho_M = 0.4\,\mathrm{fm}^{-3}$ and a droplet radius of $R = 3.0\,\mathrm{fm}$. The plasma frequency is an important quantity characterizing the lattice, because the degeneracy parameter $\eta = \hbar\omega_p/k_BT$ determines the role quantum effects play and, ultimately, the freeze-out of the OCP. In the present case, for "typical" protoneutronstar evolution temperatures, $T \sim 10\,\mathrm{MeV}$, we get for the degeneracy parameter $\eta = (\hbar Ze)/(\sqrt{M\epsilon_0}k_BTa^{3/2}) \sim 0.5$. The problem at hand can therefore be treated neither in the zero temperature- (quantum-) nor in the classical limit.

The Lindemann parameter γ^2 is defined as the value of the quantity $\langle u^2\rangle/d^2$ at the solid-liquid transition. For an OCP, it has been determined using Monte Carlo - simulations in both the classical (high temperature) limit [26, 27, 28] and in the quantum case (zero temperature) [29], for both fermions and bosons. Energetically, spin pairing effects on the droplets will drive their total spin to zero. I therefore treat them as bosons. For the intermediate degeneracies given here, the interpolation formula for the Lindemann parameter of a bosonic OCP by Chabrier [25] is used:

$$\gamma(\eta) = \gamma_0 - \frac{0.096 + 4.31\times10^{-3}\eta^2}{1+0.05\eta^2+2.092\times10^{-4}\eta^4}, \quad (7)$$

with $\gamma_0 = 0.249$ being the quantum limit. The melting curves for different charge densities on the strange droplets are plotted in figure 1, where the curves represent charge densities from $\rho_C = 0.1\,\mathrm{fm}^{-3}$ to $\rho_C = 0.8\,\mathrm{fm}^{-3}$ in steps of $\rho_C = 0.1\,\mathrm{fm}^{-3}$ for a mass density of $\rho_M = 0.4\,\mathrm{fm}^{-3}$ and droplet radii $R = 3.0\,\mathrm{fm}$. If the droplets are treated as fermions, the result only differs significantly for coexistence curves with transition temperatures below $1\,\mathrm{MeV}$. It can be seen that an initially liquid phase transition lattice crystallizes for PNS temperatures of $T \sim 1-10\,\mathrm{MeV}$, which lies well within the range obtained during the Kelvin-Helmholtz cooling timescales in the standard PNS paradigm, for a wide range of parameters (χ and ρ_C). There is, for any given charge density, a lower limit on the droplet radius below which there is no crystallization. For example for $\rho_C = 0.4\,\mathrm{fm}^{-3}$, this limit is between $1.5-1.75\,\mathrm{fm}$, see figure 2. Hence, lattices with larger droplet radii solidify earlier in the PNS evolution, causing the mixed layer to freeze out from its interior outwards.

B. Deformation Modes

The OCP assumes negative, inherently rigid, point charges in a sea of positive background. However, since the surface tension is only $\sigma \sim 10\,\mathrm{MeV/fm^2}$, small compared to strong interaction energy scales $\epsilon_{strong} \approx 10^3\,\mathrm{MeV/fm^3}$ for the given droplet dimensions, it is clear

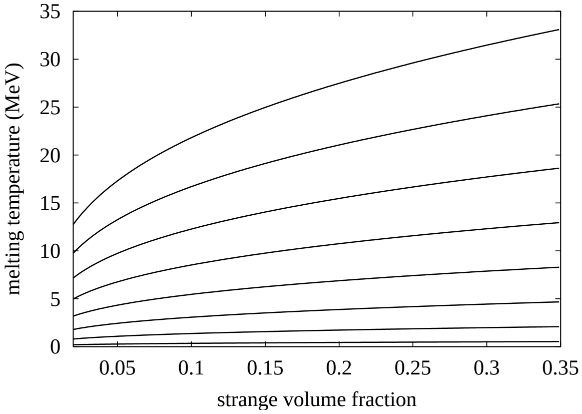

FIG. 1: Melting curve for different droplet charge densities: $\rho_C = 0.1\,\mathrm{fm}^{-3}$ (bottom curve) to $\rho_C = 0.8\,\mathrm{fm}^{-3}$ (top curve) with $\rho = 0.4\,\mathrm{fm}^{-3}$ and $R = 3.0\,\mathrm{fm}$.

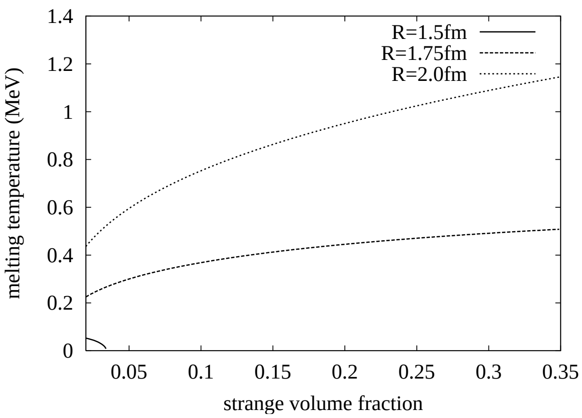

FIG. 2: Melting curve for different droplet radii with $\rho_C = 0.4\,\mathrm{fm}^{-3}$ and $\rho = 3.0\,\mathrm{fm}^{-3}$.

that the strange phase droplets cannot necessarily be considered as rigid. It is therefore important to know whether deformation modes have to be taken into consideration in the treatment of lattice vibrations.

Consider a droplet of strange matter which is slightly elongated along the x-direction to $R + dR$, yielding an ellipsoid:

$$(a, b, b) \sim (R + dR, R/\sqrt{1 + \frac{dR}{R}}, R/\sqrt{1 + \frac{dR}{R}}). \tag{8}$$

Its surface area is

$$S = 2\pi(b^2 + \frac{a^2 b}{\sqrt{a^2 - b^2}} arcsin(\frac{a^2 - b^2}{a^2})), \tag{9}$$

which, when expanded to second order in dR, yields

$$S \simeq 4\pi R^2 + 2\pi\frac{9}{8}dR^2 = S_0 + \Delta S, \tag{10}$$

from which the elastic constant k_S for the deformation energy can be deduced:

$$\Delta E_S = \sigma\Delta S = 2\pi\frac{9}{8}\sigma dR^2 = \frac{1}{2}k_S dR^2. \tag{11}$$

The inertial term m_S can be found via the kinetic energy

$$\int_{-R}^{R} dx \int_0^{\sqrt{R^2 - x^2}} d\rho(2\pi\rho[\frac{1}{2}\rho_M x^2(\frac{\omega_S}{2\pi})^2])$$
$$= \frac{1}{30\pi}\rho_M\omega_S^2 R^5 = \frac{1}{2}(\frac{4\pi}{15}\rho_M R^3)\dot{R}^2 = \frac{1}{2}m_S\dot{R}^2. \tag{12}$$

The characteristic vibration energy can therefore be estimated as

$$\omega_S = \sqrt{\frac{k_S}{m_S}} = \sqrt{\frac{9\pi\sigma/2}{M/5}}. \tag{13}$$

A typical value, for $\rho_M = 0.4\,\mathrm{fm}^{-3}$, $R = 2\,\mathrm{fm}$ and $\sigma = 10\,\mathrm{MeVfm}^{-2}$, is $\hbar\omega_S \sim 10\,\mathrm{MeV}$, which is comparable to the plasma frequency. *Hence, deformation modes freeze out at about the same temperature as lattice vibrations.* The OCP can therefore be considered as a valid description of the melting curve of the phase transition lattice, since no other modes are relevant once it becomes a crystal. At temperatures above the transition between a liquid and a solid strange droplet phase, the lattice- and deformation modes will be in thermal equilibrium.

III. SHEAR STRESSES AND THE STRANGE PHASE TRANSITION LATTICE

Besides thermodynamic criteria for the existence of a crystalline mixed phase, we need to know whether the lattice can withstand typical shear stresses present in convective PNS cores. The shear constant of a cubic Coulomb lattice is [30]

$$c_{44} = \frac{d^2 W_l}{d\gamma_{xy}^2}, \tag{14}$$

where W_l is the lattice energy and γ_{xy} the angle of distortion. The Coulomb lattice energy can be calculated using Ewald's method [31, 32]:

$$W_l = \frac{1}{2}\sum_l \frac{e^2}{4\pi\epsilon_0 r(l)} = \frac{1}{2}\frac{e^2}{4\pi\epsilon_0}\left(\sum_l \frac{\mathrm{erfc}(gr(l))}{r(l)} + \sum_l \frac{4\pi}{\Omega}\frac{exp(-G_l^2/4g^2)}{G_l^2}\right), \tag{15}$$

where the complementary error function $\mathrm{erfc}(x) \equiv \frac{2}{\sqrt{\pi}}\int_x^\infty exp(-y^2)dy$, Ω is a unit cell volume and g is a parameter to be adjusted for fast numerical convergence of both real (vectors $\vec{r}(l)$) and inverse (vectors $\vec{G}(l)$) lattice sums. The result for a bcc-lattice is [30]

$$\begin{aligned} c_{44} &= 0.7423 \times \frac{\frac{4}{3}\pi R^3 \rho_C e^2}{4\pi\epsilon_0 a} \\ &= 4.477\,\mathrm{MeV} \times (R^3\rho_C)(\frac{a}{1.0\,\mathrm{fm}})^{-1}, \end{aligned} \tag{16}$$

a being the lattice constant. The critical shear stress is the force per unit area necessary to maintain two planes of the crystal distorted against each other by an angle corresponding to a displacement of $a/4$ perpendicular to a lattice plane [33], which is in the linear approximation

$$\begin{aligned} \sigma_{crit} &= \frac{1}{NA}\frac{dU}{dx} \approx \frac{1}{A}\frac{d}{dx}(2c_{44}(\frac{x}{a})^2) \\ &= \frac{c_{44}}{a^3} = 4.477\,\mathrm{MeV} \times (R^3\rho_C)(\frac{a}{1.0\,\mathrm{fm}})^{-4}, \end{aligned} \tag{17}$$

which, for a set of typical values, a lattice constant $a = 10\,\mathrm{fm}$, droplet radius $R = 2.0\,\mathrm{fm}$ and charge density $\rho_C = 0.4\,\mathrm{fm}^{-3}$, gives $\sigma_{crit} = 1.4 \times 10^{-3}\,\mathrm{MeVfm}^{-3}$. Stresses in that order of magnitude might be caused by convection or differential rotation of the newly formed PNS. These stresses can either prevent the formation of the solid lattice in the first place or, if they are due to phenomena which are prone to variations, such as convection, break up a solid lattice formed during a transient period of weak convection.

A negative gradient in the lepton concentration has been shown to lead to convection during the Kelvin-Helmholtz cooling phase of the PNS [34]. More recently, hydrodynamics simulations including convection indicate that the Ledoux criterion for convective instability

$$C_L \equiv \left(\frac{\delta\rho}{\delta S}\right)_{P,Y_l}\frac{dS}{dr} + \left(\frac{\delta\rho}{\delta Y_l}\right)_{P,S}\frac{dY_l}{dr} > 0 \tag{18}$$

is true in most of the PNS for times of more than $\sim 1\,\mathrm{s}$ after bounce [35]. The pure strange phase in the center of the PNS, if it exists, is not expected to show strong negative lepton or entropy gradients. This is due to the relatively opaque mixed phase enclosing it and the fact that the transport of both heat and lepton number in the PNS interior is dominated by neutrinos. The most violent convection will therefore take place in the matter exterior to the phase transition lattice. Although various authors disagree on the extent and strength of convection in PNS's, convective velocities of $v_c \sim 10^6\,\mathrm{ms}^{-1}$ are reported in many studies [35, 36, 37]. This is equivalent to a kinetic energy density of $E_{conv}/V \sim 10^{-3}\,\mathrm{MeV/fm}^3$, which indicates that convection might indeed be able to either break an existing phase transition lattice or prevent its formation. A strong enough convective cell forming outside the mixed strange/non-strange layer after its crystallization during a transient quiet period might fracture the solid lattice and mix matter from the non-strange envelope into the now liquid phase transition lattice in the region below the convective cell. For the duration of the convective flow, this would result in a localized hole with a substantially lowered neutrino opacity [19] compared to the still intact solid lattice in all other directions, hence in anisotropic neutrino transport through the mixed strange/non-strange layer.

The discovery of a number of millisecond pulsars in recent years [38] indicates that some of the angular momentum residing in the core collapse supernova might remain in the PNS. The resulting rotation is likely to be differential and has been studied by Goussard et al. [39, 40] who solved the relativistic stellar structure equations for rotating PNS's with representative equations of state for the different epochs in the PNS evolution. The rotation period Ω as a function of radius r assumed in that study is (in the Newtonian limit) [40]

$$\Omega = \frac{R_0^2\Omega_C}{R_0^2 + r^2\,sin^2(\theta)}, \tag{19}$$

with $R_0 \sim 1\,\mathrm{km}$ the characteristic scale of variation of Ω, $r\,sin(\theta)$ the distance from the rotation axis and Ω_C the central rotation period. The rotation period a PNS can acquire without additional accretion has been shown to be limited by the increase of the minimum neutron star mass for times up to $\sim 100\,\mathrm{ms}$ post-bounce and by the mass shedding limit beyond that, resulting in $P_{min} \approx 1.7\,\mathrm{ms}$ [40]. This corresponds to velocities of $v \sim (r/1\,\mathrm{km})(\Omega/\Omega_C)10^6\,\mathrm{ms}^{-1}$, which is comparable in magnitude to convective velocities sufficient to cause critical shear stresses. It is not likely that regions of a fast rotating PNS at several seconds post-bounce go through a transient phase with low rotation period during which the phase transition lattice could crystallize, possibly to be broken up at later times by a larger gradient in the rotation period. Rather, for PNS's with rotation periods below $\sim 100\,\mathrm{ms}$, the timescale for the stratification of differential rotation might be of significant importance for the melting curve of the phase transition lattice, possibly comparable to the cooling timescale. However, unless the transport of angular momentum in core collapse supernovae and in particular within the PNS is finally resolved, the melting curves presented in sec. II A only apply to

PNS's with low differential rotation.

IV. SUMMARY AND DISCUSSION

The present study shows that if the phase transition from hadronic PNS matter to strange matter is of first order and if it results in a lattice of separate strange and non-strange phases in the coexistence zone, it will crystallize for temperatures predicted during the Kelvin-Helmholtz cooling phase of a PNS following a core collapse supernova. The process of crystallization has, however, complex interactions with the hydrodynamical evolution of the PNS. Shear stresses due to strong differential rotation with minimum periods in the range observed for millisecond pulsars and convection induced by lepton gradients might, as long as they persist, prevent the formation of a solid lattice. A final answer can only be given by the full inclusion of a possibly solid mixed strange/non-strange phase in future core collapse supernova simulations.

If the first-order phase transition occurs before the end of the Kelvin-Helmholtz cooling phase, the remaining neutrinos to be emitted might serve as a possible window on its formation and crystallization. With the predicted increase in the neutrino opacity for a first order phase transition lattice [19], expected to be especially pronounced for intermediate neutrino energies in the range $\sim 10 - 100\,\mathrm{MeV}$, the timescale for the decay of the neutrino emission will increase, which might be observable as a knee in the neutrino luminosity. Once the lattice becomes solid, the evolution might be accompanied by fractures and rearrangements in the mixed strange/non-strange zone, possibly showing irregularities in the neutrino luminosity. If a changing lepton number gradient causes a convective cell to form in a formerly non-convective region located outside a previously crystallized phase transition lattice, sufficiently violent convection might break the solid lattice and locally cause it to return to a liquid state. Non-strange PNS matter transported from regions outside the phase transition lattice might mix with the liquid, resulting in an anisotropic neutrino-opaque layer with a partial hole and therefore anisotropic neutrino transport. This will be more pronounced if the central density of the PNS is high enough to allow for a pure strange core enclosed by the relatively opaque mixed zone which essentially dams up neutrinos behind it. The extent to which the phase transition lattice will affect the neutrino luminosity and emission spectrum will be the topic of a detailed transport study with a special focus on anisotropic neutrino transport in conjunction with a hydrodynamical treatment of the non-strange layers above the coexistence zone [41]. This will also include the different neutrino transport properties of a solid versus a liquid lattice.

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